Note on an interesting metric of the field equations in general relativity

1968 J. Phys. A: Gen. Phys. 2 138

(http://iopscience.iop.org/0022-3689/2/1/019)

is the above-mentioned equation (1). This general equation is expressed as

$$W = 1 - \int_0^{-\infty} \eta(\Omega) \exp\left(- \int_{T_0}^{T} w\Omega \, dT \right) d\Omega \tag{5}$$

and corresponds to the case when the polydispersional aerosol is crystallized, all its drop sizes being characterized by its own cooling rate.

Equations (3) were repeatedly used to calculate the crystallization of the aerosol (see, for example, Kachurin 1959).

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J. PHYS. A (GEN. PHYS.), 1969, SER. 2, VOL. 2. PRINTED IN GREAT BRITAIN

Note on an interesting metric of the field equations in general relativity

Abstract. In the course of investigating the Rainich equations of the 'already unified field theory' in the case when the electromagnetic field is non-static, Bera and Datta in 1968 obtained a metric which leads to empty flat space. It is shown that the said metric is Riemannian (non-flat) and satisfies the field equations of gravitation for empty space. Hence it is of great interest from the point of view of cosmology.

In the course of investigating the Rainich equations of the 'already unified field theory' in the case when the electromagnetic field is non-static and the space–time metric admits a group G_4 of automorphisms, Bera and Datta (1968) have very recently obtained the metric

$$ds^2 = (dx^4)^2 - B(kx^4 + l)^{4/3}\{(dx^1)^2 + (dx^2)^2\} - \frac{16AB}{9}(kx^4 + l)^{-2/3}(dx^3)^2 \tag{1}$$

for which

$$R_4{}^4 = 0. \tag{2}$$

Here x^4 is the time coordinate and x^1, x^2, x^3 are the space coordinates.

We note in passing that the metric (1) admits an intransitive group of motions and that the group G_4 includes the Abelian subgroups G_3.

The metric is seen to satisfy the field equations of gravitation for empty space

$$G_{\mu\nu} = 0 \tag{3}$$

and, moreover, the pseudo-tensor density of gravitational energy and momentum vanishes everywhere, i.e.

$$t_\mu{}^\nu = 0. \tag{4}$$

By straightforward calculation one can see, for the metric (1), that the only surviving components of curvature tensor $R_{\alpha\beta\gamma\delta}$ are

$$\left.\begin{aligned} R_{4114} &= R_{4224} \\ &= \tfrac{2}{9}Bk^2(kx^4+l)^{-2/3} \\ R_{4334} &= -\tfrac{64}{81}ABk^2(kx^4+l)^{-8/3} \\ R_{2112} &= \tfrac{4}{9}B^2k^2(kx^4+l)^{2/3} \\ R_{3113} &= R_{3223} \\ &= -\tfrac{32}{81}AB^2k^2(kx^4+l)^{-4/3} \end{aligned}\right\}. \tag{5}$$

Thus the metric under consideration, although it leads to empty space, is Riemannian (non-flat) by virtue of (5). Hence the conclusion arrived at by Bera and Datta (1968) that the metric leads to empty flat space is not correct.

By considering the metric in the more general form

$$ds^2 = (dx^4)^2 - (kx^4+l)^m(dx^1)^2 - (kx^4+l)^n(dx^2)^2 - (kx^4+l)^p(dx^3)^2 \tag{6}$$

one can easily obtain for gravitational fields in empty space

$$\left.\begin{aligned} m = n &= \tfrac{4}{3} \\ p &= -\tfrac{2}{3} \end{aligned}\right\} \tag{7}$$

which yields the same solution as obtained earlier.

While studying static gravitational fields in general relativity, Das (1968) has, of late, obtained a conformastat gravitational universe

$$ds^2 = (1-mx^1)^{-2}(dx^4)^2 - (1-mx^1)^4\{(dx^1)^2+(dx^2)^2+(dx^3)^2\} \tag{8}$$

where m is a constant and $1-mx^1$ is the potential function. The above metric is due to 'an infinite plate parallel to the (x^2, x^3) plane', which gives via an 'illegal' transformation the non-static metric

$$ds^2 = (dx^4)^2 - (x^4)^{-2/3}(dx^1)^2 - (x^4)^{4/3}\{(dx^2)^2+(dx^3)^2\} \tag{9}$$

$x^4 > 0$. This is the same metric as has been obtained by Bera and Datta (1968). From the point of view of cosmology, the above metric is of great interest and deserves consideration. One may imagine that the field given by the metric represents a transitional model, like a vacuum pocket into which matter is introduced from the surrounding portions of an extragalactic nebula.

The author wishes to express his gratitude to Professor B. K. Datta, Surendranath College, Calcutta, India, for his guidance and help, and thanks Mr. N. De, Jadavpur University, Calcutta, India, for his valuable suggestion.

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14th August 1968

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